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Fairness in Missing Data Imputation

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Abstract

Missing data are ubiquitous in the era of big data and, if inadequately handled, are known to lead to biased findings and have deleterious impact on data-driven decision makings. To mitigate its impact, many missing value imputation methods have been developed. However, the fairness of these imputation methods across sensitive groups has not been studied. In this paper, we conduct the first known research on fairness of missing data imputation. By studying the performance of imputation methods in three commonly used datasets, we demonstrate that unfairness of missing value imputation widely exists and may be associated with multiple factors. Our results suggest that, in practice, a careful investigation of related factors can provide valuable insights on mitigating unfairness associated with missing data imputation.

1. Introduction

Missing data, as well-recognized, have significant impact on analysis of real data in many fields. One of the most popular approaches for handling missing data is missing data imputation. With the development of statistics and machine learning, different approaches have been adopted for the imputation task (Gondara & Wang, 2018; Li et al., 2019; Stekhoven & Bühlmann, 2012; Van Buuren & Oudshoorn, 1999; Yang et al., 2019; Yoon et al., 2018). While these machine learning imputation methods show preferable performances, potential concern about unfairness of the algorithms is non-negligible. Machine learning algorithms have been shown to inherit the bias and unfairness that human have in decision making. Various studies are conducted in the context of computer vision and natural language processing (Bolukbasi et al., 2016; Brunet et al., 2019; Buolamwini & Gebru, 2018; Gonen & Goldberg, 2019; Klare et al., 2012; Ryu et al., 2017; Zhang & Liu, 2020; Zhao et al., 2018; 2019b), illustrating the wide existence of unfairness and possible remedies. Algorithmic fairness, focusing on non-discrimination of decision outcomes, comes to the fore in the research community.

As discussed in Chouldechova & Roth (2018), Martínez-Plumed et al. (2019) and Pessach & Shmueli (2020), the causes of unfairness in machine learning mainly come from bias in dataset, missing values, prediction algorithms as well as imbalance of populations among different sensitive groups. In particular, it's discussed in Bakker et al. (2020), Martínez-Plumed et al. (2019), Rajkomar et al. (2018) and Gianfrancesco et al. (2018) that missing values contribute to the bias of algorithms. Empirical analysis in Martínez-Plumed et al. (2019) brings to the fore the issue of correctly handling missing data in the sense of fairness, instead of dropping out corresponding samples directly. Inspired by this work, we study the fairness of some representative imputation methods in three datasets*: COMPAS recidivism, Alzheimer's Disease Neuroimaging Initiative (ADNI) and atherosclerosis cardiovascular disease (ASCVD). By looking into imputation results and corresponding predictions for each imputation method and missing data mechanism, we observe non-negligible bias among different gender and race groups among all the imputation methods compared.

In the context of the impact missing data have on fairness, Martínez-Plumed et al. (2019) studies fairness of existing real datasets with missing values; Wang & Singh (2021) studies the impact of missing values in categorical data on fairness; Goel et al. (2020) illustrates the recoverability of outcome's distribution from complete cases, through a causal modeling of data missingness. Our work is the first work to systematically study the fairness associated with the process of imputing missing data.

Our contributions: In this paper, we propose the first fairness notion in imputation, *imputation accuracy parity dif-ference*. We also study fairness of prediction model built on imputated datasets. Our empirical studies show that (1) Severe unfairness exists in both imputation and prediction after imputation. (2) Different imputation methods have non-negligible impact on fairness. (3) Unfairness in both imputation and prediction can be associated with the sample

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^{*}A detailed description of the datasets can be found in Appendix A

imbalance and missing data mechanism. Our work provides
valuable insight into how to effectively handle missing values while guaranteeing fairness of the learning tasks.

2. Preliminaries

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2.1. Fairness notions

062 Discussion on fairness are based on pre-specified sensi-063 tive attributes A. In real data experiments, we use gen-064 der and ethnicity as sensitive attributes, which are widely 065 used in social science study. For gender, we identify 066 male as the majority group and female as the minority 067 group. For ethnicity we identify white as the major-068 ity group and black (or "other races") as the minority 069 group. Regarding the fairness notions, Equalized Odds 070 (EO) (Hardt et al., 2016) is widely used. For a deci-071 sion making procedure, EO aims to let algorithm's prediction \hat{y} independent of sensitive attribute A, conditioned 073 on the true outcome y. Consider the learning task in which 074 data Z contains predictor X and response y. When re-075 sponse y is binary, false positive rate (FPR) and false neg-076 ative rate (FNR) for classifier h in group a are defined as 077 $\begin{aligned} & \operatorname{FPR}_{a}(h) := \sum_{A_{i}=a} \mathbf{1}\{h(x_{i}) = 1\} / \sum_{A_{i}=a} \mathbf{1}\{y_{i} = 0\} \\ & \operatorname{and} \operatorname{FNR}_{a}(h) := \sum_{A_{i}=a} \mathbf{1}\{h(x_{i}) = 0\} / \sum_{A_{i}=a} \mathbf{1}\{y_{i} = 0\} \end{aligned}$ 078 079 1}. We define equalized odds difference as the fairness 080 notion in classification: 081

Definition 2.1 (equalized odds difference) Equalized odds difference for classifier h is defined as EOD(h) := $|FPR_{maj}(h) - FPR_{min}(h)| + |FNR_{maj}(h) - FNR_{min}(h)|.$

Here the subscripts "maj" and "min" mean the quantities are for majority and minority sensitive groups, respectively. The notion is a measure of how close the prediction algorithm his to the equalized odds, in which EOD equals to 0.

091 In addition, in this paper we propose a novel notion that 092 measures the fairness of imputation results. We narrow our 093 scope to the scenario when sensitive attributes are binary 094 $A = a \in \{0, 1\}$. Suppose the complete data matrix, without missing values, is denoted by $\mathbf{Z} = (z_{ij}) \in \mathbb{R}^{n \times p}$, and 095 the missing indicator is denoted by $\mathbf{R} = (r_{ij}) \in \mathbb{R}^{n \times p}$, 096 with $r_{ij} = \mathbf{1}\{z_{ij} \text{ is observed}\}$. Let $\mathbf{Z}_{obs} = \{z_{ij} | r_{ij} = 1\}$ 097 098 denote the observed data and $\mathbf{Z}_{\text{miss}} = \{z_{ij} | r_{ij} = 0\}$ denote the missing data. We further let $\mathbf{Z}^a = (z_{ij}^a)$ and $\mathbf{R}^a =$ 099 100 (r_{ij}^a) denote the complete data matrix and missing indicator matrix in sensitive group A = a (so that $\mathbf{Z} = \mathbf{Z}^0 \bigcup \mathbf{Z}^1$, $\mathbf{R} = \mathbf{R}^0 (\mathbf{J} \mathbf{R}^1)$. In group *a*, the data matrix imputed by model g is denoted by $\widehat{\mathbf{Z}}^{a}(g) = (\widehat{z}^{a}_{ij}(g))$. Assume that 104 both sensitive groups contain missing data, we define mean 105 square imputation error of q in group a as

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$$MSIE_{a}(g) = \frac{\sum_{(i,j)} (\hat{z}_{ij}^{a}(g) - z_{ij}^{a})^{2} (1 - r_{ij}^{a})}{\sum_{(i,j)} 1 - r_{ij}^{a}}$$

Now we define *imputation accuracy parity*, a novel notion that measures fairness of imputation model *g*:

Definition 2.2 (imputation accuracy parity difference)

Imputation accuracy parity for imputation model g is defined as $IAPD(g) = MSIE_{maj}(g) - MSIE_{min}(g)$.

Imputation accuracy parity is similar to the fairness notion *accuracy parity* adopted in multiple literature (Friedler et al., 2016; Zafar et al., 2017; Zhao et al., 2019a). Consider the learning tasks where data \mathbf{Z} contains predictor X and binary response y. When only y contains missing values, imputation can be regarded as a prediction task. In such case, accuracy parity for imputation method g can be regarded as a finite-sample version of the accuracy parity.

2.2. Missing data mechanism

The missing data mechanism (Little & Rubin, 2019) can be classified into three types: missing completely at random (MCAR), missing at random (MAR) and missing not at random (MNAR). A missing mechanism is said to be MCAR if missingness is independent of both observed and missing data. When missingness is only dependent on observed data, the mechanism is said to be MAR. For the MNAR, missingness can be associated with both observed and unobserved data. In this paper, we conduct real data experiments to assess fairness associated with imputation in which missing data are artificially generated under all three mechanisms. Specifically, for each real data set used, we normalize all the features and then generate missing values in the first L features in the dataset, with a pre-specified L. Throughout, we use the following 11 models for generating missing values. Given a sample $\mathbf{z} = (z_1, \cdots, z_p)$, the probability that z_i is missing is given by the following values[†], for $\forall j \in \{1, \ldots, L\}$:

MCA	AR	MAR		MNAR							
0.1	(1a)	$0.1 + 0.8 1_{male}$	(2a)	$0.5 - z_{j}$	(3a)						
0.5	(1b)	$0.1 + 0.8 1_{\mathrm{female}}$	(2b)	$0.5 - 0.2z_j$	(3b)						
0.9	(1c)	$0.5 - 0.5 z_{L+j}$	(2c)	$0.5 + 0.2z_j$	(3c)						
		$0.5 + 0.5 z_{L+i}$	(2d)	$0.5 + z_i$	(3d)						

2.3. Imputation methods

The imputation methods investigated include MICE (Buuren & Groothuis-Oudshoorn, 2010), missForest (Stekhoven & Bühlmann, 2012), K-nearest neighbor (KNN) imputation, two matrix completion methods SoftImpute (Hastie et al., 2015) and OptSpace (Keshavan et al., 2010), and also two deep learning methods Gain (Yoon et al., 2018) and Misgan (Li et al., 2019). A more thorough review on existing imputation methods is provided in Appendix B.

[†]the values are truncated inside the unit interval [0, 1]

Fairness in Missing Data Imputation

MSIE									IAPD regarding gender										IAPD regarding race								
Method	$ I_1 $	I_2	I_3	I_4	I_5	I_6	I_7	Var	I_1	I_2	I_3	I_4	I_5	I_6	I_7	$VarD_g$	I_1	I_2	I_3	I_4	I_5	I_6	I_7	VarD _r			
MCAR (1a)	0.68	0.68	0.67	0.81	0.88	0.73	0.99	0.99	0.45	0.46	0.47	0.50	0.48	0.36	0.51	0.55	-0.46	-0.48	-0.48	-0.49	-0.53	-0.32	-0.50	-0.53			
MCAR (1b)	0.90	0.82	0.80	0.91	0.87	0.86	0.95	1.00	0.58	0.51	0.51	0.52	0.50	0.42	0.52	0.56	-0.57	-0.56	-0.55	-0.52	-0.54	-0.42	-0.49	-0.54			
MCAR (1c)	0.85	1.09	0.88	0.96	0.88	0.95	0.95	1.00	0.54	0.58	0.53	0.53	0.51	0.44	0.47	0.56	-0.55	-0.73	-0.59	-0.53	-0.55	-0.43	-0.46	-0.54			
MAR (2a)	0.89	1.08	0.85	1.03	0.90	1.00	1.04	1.09	0.53	0.79	0.57	0.56	0.50	0.60	0.50	0.55	-0.63	-0.76	-0.64	-0.61	-0.62	-0.51	-0.57	-0.61			
MAR (2b)	0.47	0.53	0.49	0.62	0.57	0.70	0.69	0.72	0.43	0.35	0.41	0.49	0.49	0.14	0.51	0.55	-0.23	-0.27	-0.25	-0.24	-0.28	-0.15	-0.22	-0.27			
MAR (2c)	0.42	0.36	0.54	0.51	0.53	0.80	0.62	0.57	0.15	0.15	0.13	0.18	0.19	0.11	0.17	0.21	-0.12	-0.22	-0.04	-0.21	-0.23	-0.04	-0.06	-0.25			
MAR (2d)	1.27	1.66	1.35	1.36	1.34	1.37	1.48	1.44	0.86	0.85	0.92	0.83	0.79	0.73	0.84	0.83	-0.72	-0.87	-0.81	-0.64	-0.60	-0.65	-0.77	-0.62			
MNAR (3a)	1.08	1.16	2.44	0.26	0.36	0.65	0.64	0.12	0.23	0.21	0.14	0.00	-0.01	0.00	0.01	0.01	-0.35	-0.47	-0.30	0.03	0.02	0.01	0.08	0.02			
MNAR (3b)	0.67	0.53	0.67	0.28	0.35	0.60	0.52	0.27	0.25	0.16	0.14	0.02	0.01	-0.01	0.03	0.05	-0.27	-0.32	-0.29	0.00	0.03	0.02	-0.05	-0.03			
MNAR (3c)	1.50	1.42	1.62	1.59	1.43	1.67	1.85	1.59	1.09	1.06	1.14	1.04	1.00	0.98	1.04	0.95	-1.01	-1.02	-1.13	-1.01	-1.01	-0.87	-0.97	-0.89			
MNAR (3d)	2.71	2.44	2.53	2.20	1.88	1.95	2.11	1.71	1.59	1.47	1.56	1.35	1.28	1.01	1.03	1.00	-1.18	-1.32	-1.25	-1.20	-1.20	-0.83	-0.88	-0.90			

Table 1. Imputation fairness on COMPAS recidivism dataset. Number of features (besides sensitive attributes) is 10, L = 5. Here imputation methods are encoded as: I_1 : MICE; I_2 : missForest; I_3 : KNN; I_4 : SoftImpute; I_5 : OptSpace; I_6 : Gain; I_7 : misGAN. Results are average values over 50 repeated experiments. Var denotes the variance of missing data, $VarD_q$ denotes the difference of missing value's variances between two gender groups and $VarD_r$ denotes that between two race groups.

3. Fairness in imputation accuracy

129 In this section, we investigate the imputation fairness for dif-130 ferent existing methods under various missing mechanisms. 131 Each experiment is repeated for 50 times and results shown 132 are the average values. Let $VarD_q$ and $VarD_r$ denote the 133 difference of missing value's variances between two gender 134 groups and between two racial groups, respectively. Both of 135 the quantities serve as baselines of imputation performance, 136 since the variance difference is equivalent to IAPD when 137 using imputing missing values with mean observed values in 138 two groups respectively. We set L = 5 in the experiment of 139 COMPAS dataset and report the results in Table (1). Results 140 for two other real datasets are provided in Appendix C and 141 D.

143 Observation 1: Severe imputation unfairness widely ex-144 ists

We observe that in all the 11 missing mechanisms, almost 146 all the imputation methods have positive imputation IAPD 147 regarding gender and negative IAPD regarding race. This 148 implies that regarding gender, all the imputation methods 149 consistently provide more accurate imputation result for fe-150 male group compared with male group. Meanwhile, all the 151 imputation methods give more accurate imputation result 152 for white people compared with black people. Severe impu-153 tation unfairness are observed in this dataset among all the 154 imputation models. In Appendix C and D, we also observe 155 imputation unfairness in ADNI and ASCVD datasets. 156

157 **Observation 2: Imputation fairness can be influenced** 158 by imbalance of missingness

The missingness in MAR (2a) and (2b) are approximately 160 0.5, which is in the similar level as MCAR (1b). However, in 161 mechanism (2a), about 90% of first 5 features in male group 162 are missing while only 10% of that in female group are 163

missing. This difference can cause significant influence on imputation fairness: The more missingness one group has, the larger imputation error it appears to have. From Table 1, for many imputation methods we found that imputation IAPD regarding gender in MAR (2a) is consistently larger (taking sign into account) than that in mechanism MCAR (1b). This indicates male group's imputation error becomes relatively larger in MAR (2a). Meanwhile, all the imputation methods gives smaller imputation IAPD regarding gender in MAR (2b) than that in mechanism MCAR (1b). These empirical observation indicates that for a fixed overall missingness, imbalance of missingness between two sensitive groups can also influence imputation fairness effectively. Additional evidence is also observed in the experiments of ADNI dataset, shown in Appendix C.

Observation 3: Imputation unfairness tends to be enlarged as missingness increases

Among 3 MCAR mechanisms, from (1a) to (1c) the missingness increases (from 0.1 to 0.9). A trend of increasing imputation unfairness (IAPD regarding gender and race) is also observed for most imputation methods. This implies that degree of missingness contributes to the imputation fairness. Intuitively, the bias in imputation can be amplified by increasing missingness.

Besides, in Appendix C we have an additional observation that imputation unfairness can be associated with imbalance of sample size. Of all 649 patients in ADNI gene dataset, 642 patients are white people. We observe that the IAPD regarding race groups is almost always negative, implying imputation methods' preferences towards white people during imputation. This effect of sample imbalance is also observed in ASCVD dataset (Appendix D), where population in white group is two times that in the other group and that IAPD regarding race groups is also consistently

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	Prediction Accuracy								EOD regarding gender										EOD regarding race							
Method	I_1	I_2	I_3	I_4	I_5	I_6	I_7	CC	I_1	I_2	I_3	I_4	I_5	I_6	I_7	CC_g	I_1	I_2	I_3	I_4	I_5	I_6	I_7	CC_r		
MCAR (1a)	0.71	0.71	0.71	0.63	0.65	0.60	0.71	0.72	0.18	0.16	0.16	0.05	0.06	0.06	0.17	0.16	0.29	0.29	0.29	0.03	0.05	0.04	0.30	0.29		
MCAR (1b)	0.68	0.70	0.68	0.63	0.65	0.58	0.70	0.71	0.17	0.16	0.14	0.05	0.06	0.03	0.17	0.16	0.27	0.29	0.26	0.03	0.04	0.05	0.31	0.29		
MCAR (1c)	0.67	0.66	0.66	0.64	0.65	0.64	0.64	0.70	0.17	0.13	0.13	0.06	0.06	0.08	0.12	0.15	0.23	0.24	0.23	0.04	0.07	0.15	0.20	0.29		
MAR (2a)	0.69	0.69	0.69	0.64	0.65	0.63	0.69	0.70	0.14	0.15	0.13	0.06	0.05	0.06	0.14	0.26	0.24	0.24	0.25	0.03	0.04	0.11	0.24	0.27		
MAR (2b)	0.70	0.70	0.71	0.64	0.65	0.61	0.70	0.71	0.17	0.15	0.16	0.06	0.05	0.05	0.17	0.16	0.31	0.30	0.29	0.04	0.03	0.04	0.30	0.29		
MAR (2c)	0.46	0.46	0.46	0.64	0.65	0.52	0.46	0.71	0.01	0.00	0.02	0.06	0.06	0.03	0.03	0.16	0.02	0.00	0.02	0.03	0.07	0.15	0.06	0.30		
MAR (2d)	0.57	0.61	0.59	0.54	0.65	0.50	0.54	0.71	0.01	0.06	0.03	0.00	0.07	0.01	0.01	0.17	0.00	0.05	0.02	0.00	0.05	0.10	0.03	0.29		
MNAR (3a)	0.68	0.66	0.62	0.64	0.66	0.61	0.64	0.69	0.12	0.12	0.09	0.05	0.06	0.04	0.01	0.14	0.32	0.24	0.16	0.03	0.35	0.03	0.08	0.25		
MNAR (3b)	0.67	0.69	0.69	0.64	0.65	0.62	0.67	0.71	0.12	0.16	0.16	0.05	0.08	0.05	0.06	0.16	0.26	0.28	0.26	0.03	0.17	0.04	0.16	0.29		
MNAR (3c)	0.70	0.69	0.68	0.64	0.65	0.57	0.69	0.71	0.17	0.13	0.12	0.06	0.06	0.03	0.09	0.16	0.31	0.26	0.25	0.04	0.03	0.05	0.18	0.28		
MNAR (3d)	0.69	0.68	0.69	0.64	0.65	0.58	0.69	0.71	0.12	0.09	0.09	0.07	0.07	0.03	0.15	0.16	0.27	0.26	0.26	0.04	0.06	0.05	0.31	0.28		

178Table 2. Prediction fairness on COMPAS recidivism dataset. Number of features (besides sensitive attributes) is 10, L = 5. Here imputation179methods are encoded as: I_1 : MICE; I_2 : missForest; I_3 : KNN; I_4 : SoftImpute; I_5 : OptSpace; I_6 : Gain; I_7 : misGAN. CC: prediction180using complete cases in the training set. Results are average values over 50 repeated experiments. When using complete data for prediction,181prediction accuracy is 0.72, accuracy difference regarding gender is 0.16, accuracy difference regarding race is 0.29.

negative. This observation matches intuition that imputation
accuracy is positively correlated with the amount of information observed. Imbalance of observed information leads
to imputation unfairness.

4. Fairness in prediction accuracy

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189 We study the prediction fairness for different imputation 190 methods on three aforementioned datasets, each of which 191 contains a response y. In each experiment, we firstly conduct a train-test split (80 % and 20 % of the sample size, 193 respectively), artificially generate missing data in the training set, according to the 11 missing mechanisms in Section 195 2. Next, we impute the missing data using all the imputation 196 models mentioned in Section 2.3. Finally, we train a pre-197 diction model h using random forest, based on the original 198 training set (without missing values), complete cases in the 199 training set and imputed datasets. Equalized odds difference 200 (Definition 2.1) on the test set are reported. The results 201 for COMPAS is shown in Table 2 and those for other two 202 datasets are provided in Appendix C and D.

204 **Observation 4: Prediction fairness is associated with** 205 **missing mechanism**

206 We observe that for each imputation method, i.e., a fixed column, the EOD values vary cross different rows. This 208 implies that different missing mechanisms lead to different 209 prediction fairness when the imputation method is fixed. In 210 particular, in MAR (2c), all the imputation methods has 211 smaller EOD regarding both gender and race, compared 212 with the EODs when using complete data or complete cases 213 to build the prediction model. 214

Observation 5: Imputation posts a trade-off between accuracy and fairness in prediction

From Table 2, we observe that for a fixed missing mechanism, prediction fairness associated with different imputation methods are different. In particular, we observe that Gain and two matrix completion methods: SoftImpute and OptSpace consistently have smaller EOD compared with other imputation methods (and prediction models without imputation, using complete cases and complete data).

At the same time, prediction algorithms associated with these three methods (i.e., Gain, SoftImput and OptSpace) have lower prediction accuracy. This can be viewed as a trade-off between prediction accuracy and prediction fairness. In fact, we notice from the table that such trade-off widely exists. For an arbitrary missing mechanism, the prediction accuracy associated with an imputation method is lower than that associated with complete data, and most imputation models are also associated with a smaller EOD compared with that associated with the morel built through the complete data.

In addition, in the experiment of ADNI data (shown in Table 4), the EOD for race is larger than 0.15 in most cases. A potential reason is that 624 out of 649 samples are from white patients. This suggests that prediction unfairness can also be influenced by the sample imbalance in some extreme cases, including our experiments of ADNI data.

5. Discussion

In this paper we study the fairness associated with missing data imputation in three real datasets. Our experiments show that imputation unfairness widely exists among different imputation models, representing the first known empirical results in literature. We also demonstrate factors that could contribute to imputation fairness. We further study the impact of imputation on prediction fairness when imputed data are used to build prediction models. This area offers fertile ground for theoretical investigation, as there has been littler exiting work in this area.

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