Towards a Unified Framework for Fair and Stable Graph Representation Learning

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Abstract

As the representations output by Graph Neural Networks (GNNs) are increasingly employed in real-world applications, it becomes important to 015 ensure that these representations are fair and stable. In this work, we establish a key connection between fairness and stability and leverage it to 018 propose a novel framework, NIFTY (uNIfying Fairness and stabiliTY), which can be used with 020 any GNN to learn fair and stable representations. We introduce an objective function that simultaneously accounts for fairness and stability and proposes laver-wise weight normalization of GNNs using the Lipschitz constant. Further, we theoreti-025 cally show that our layer-wise weight normalization promotes fairness and stability in the result-027 ing representations. We introduce three new graph 028 datasets comprising of high-stakes decisions in 029 criminal justice and financial lending domains. 030 Extensive experimentation with the above datasets demonstrates the efficacy of our framework.

1. Introduction

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035 Over the past decade, there has been a surge of interest in leveraging GNNs for graph representation learning. GNNs are used to learn powerful representations for down-038 stream applications—e.g., predicting protein-protein interactions (Huang et al., 2020), drug repurposing (Zitnik et al., 2018), crime forecasting (Jin et al., 2020). As GNNs are 041 increasingly implemented in real-world applications, it becomes important to ensure that these models and their rep-043 resentations are safe and reliable. Specifically, ensuring that the model's resulting representations are not perpetrating 045 undesirable discriminatory biases (i.e., fairness), and are 046 robust to attacks resulting from small perturbations to the 047 graph structure and node attributes (*i.e.*, stability).



Figure 1. NIFTY can learn fair and stable representations (*i.e.*, invariant to the sensitive attribute value and perturbations to the graph structure and non-sensitive attributes) by maximizing the similarity between representations from diverse augmented graphs.

A myriad of GNN methods with various neighborhood aggregation schemes have been developed recently (e.g., Kipf & Welling (2017); Hamilton et al. (2017); Xu et al. (2018; 2019); Veličković et al. (2019)). While these methods achieve state-of-the-art performance in tasks such as node classification and link prediction, they can be prone to discrimination and instability (Dai & Wang, 2021; Rahman et al., 2019; Bose & Hamilton, 2019). Since GNNs compute node representations by propagating and aggregating neural messages along edges in graph neighborhoods, nodes with similar sensitive attribute values are likely to share similar representations leading to severe discriminatory biases. While previous techniques study fairness (Dai & Wang, 2021) and stability (Zhu et al., 2019) independently, it remains an open question whether there is any deeper connection between these properties, and if they can be achieved simultaneously.

Present work. To tackle the problem of learning fair and stable representations, we first identify a key connection between fairness and stability. While stability accounts for robustness *w.r.t.* random perturbations to node attributes and/or edges, fairness accounts for robustness *w.r.t.* modifications of the sensitive attribute. We use the above connection to develop NIFTY to enforce fairness and stability in the objective function as well as in the message passing step of the GNN layers. Results show that NIFTY improves the

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fairness and stability of five GNNs by 92.01% and 60.87%
 without sacrificing their predictive performances.

058 **2. Preliminaries**

060 Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{X})$ denote an undirected graph on nodes \mathcal{V} 061 and= edges \mathcal{E} . Let $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ denote vectors corre-062 sponding to the nodes in \mathcal{V} , where $\mathbf{x}_v \in \mathbb{R}^M$ captures node *v*'s attributes. Let $\mathbf{A} \in \mathbb{R}^{N \times N}$ be the adjacency matrix where 063 $A_{uv}=1$ if there exists some edge between nodes u and v, 064 065 and otherwise 0. We use \mathcal{N}_u to denote the immediate neighbors of node u, *i.e.*, $\mathcal{N}_u = \{v \in \mathcal{V} | A_{uv} = 1\}$. Furthermore, let 066 067 $\mathbf{I}_u \in \{0,1\}^N$ denote the incidence vector which captures all the edges incident on node u, *i.e.*, $\mathbf{I}_{uv} = 1$ if an edge exists be-068 069 tween nodes u and v, and otherwise 0. Finally, we introduce 070 \mathbf{b}_u to capture all the information associated with node u, *i.e.*, 071 $\mathbf{b}_{u} = [\mathbf{x}_{u}; \mathbf{I}_{u}]$ denotes the concatenation of node attribute and incidence vector corresponding to node u. We also gener-073 ate an augmented graph $\mathcal{G}' = (\mathcal{V}, \mathcal{E}', \tilde{\mathbf{X}}), i.e.$, for each node 074 we generate a corresponding node in the augmented graph 075 by slightly perturbing the attribute values, incident edges, 076 and/or modifying u's sensitive attribute. For a GNN with 077 K layers, the representations for node u for each layer is denoted as $\mathbf{h}_{u}^{1}, \cdots, \mathbf{h}_{u}^{K}$, where $\mathbf{z}_{u} = \mathbf{h}_{u}^{K}$ is representation at 078 the last GNN layer. Analogously, $\tilde{\mathbf{z}}_{u}$ denotes the output rep-079 resentation of node u in \mathcal{G}' . The (dis)similarity between two 081 node representations is given by a distance metric $D: \mathbb{R}^d \times$ 082 $\mathbb{R}^d \to \mathbb{R}$. Our goal is to learn an encoder ENC which maps 083 node u to its representation \mathbf{z}_u , *i.e.*, ENC(u)= \mathbf{z}_u . Lastly, a 084 classifier f maps the representation \mathbf{z}_u to a class label \hat{y}_u .

085 Graph Neural Networks. GNNs can be formulated as 086 message passing networks (Wu et al., 2020) specified by 087 trainable operators MSG, AGG, and UPD. In a K-layer 088 GNN, the operators are recursively applied on \mathcal{G} , specifying 089 how messages are exchanged between nodes, aggregated, 090 and transformed to generate final node representations. A 091 message between a pair of nodes (u, v) in layer k is defined 092 as a function of hidden node representations from the previ-093 ous layer as: $\mathbf{m}_{uv}^k = MSG(\mathbf{h}_u^{k-1}, \mathbf{h}_v^{k-1})$. In AGG, messages from \mathcal{N}_u are aggregated as: $\mathbf{m}_u^k = AGG(\mathbf{m}_{uv}^k|u \in \mathcal{N}_u)$. In 094 095 UPD, the aggregated message \mathbf{m}_{u}^{k} is combined with \mathbf{h}_{u}^{k-1} 096 to produce u's representation as: $\mathbf{h}_{u}^{k} = \text{UPD}(\mathbf{m}_{u}^{k}, \mathbf{h}_{u}^{k-1}).$ 097

Fairness and Stability. Our goal is to learn fair and stable
node representations. More specifically, the notions of
fairness and stability that we consider in this work are counterfactual fairness and Lipschitz continuity, respectively.

Counterfactual Fairness: For graph representation learning, counterfactual fairness can be interpreted as node representations output by encoders should be independent of the sensitive attribute, *i.e.*, changing node u's sensitive attribute value should not affect the node representations.

108 Definition 1. An encoder function ENC satisfies counter-

factual fairness if the following holds for any given node u:

$$\operatorname{ENC}(u) = \operatorname{ENC}(\tilde{u}^s),\tag{1}$$

where \tilde{u}^s is a node in the augmented graph generated by modifying *u*'s sensitive attribute values while keeping everything else constant.

Stability via Lipschitz Continuity: A function is stable according to Lipschitz continuity if slightly perturbing any given instance does not drastically change the output. In graph representation learning, this notion entails small perturbations to node attributes and/or incident edges should not drastically change the resulting representations.

Definition 2. An encoder function ENC is stable according to the notion of Lipschitz continuity if:

$$|\operatorname{ENC}(\tilde{u}) - \operatorname{ENC}(u)||_p \le L||\tilde{\mathbf{b}}_u - \mathbf{b}_u||_p, \qquad (2)$$

where \tilde{u} is a node in the augmented graph generated by perturbing *u*'s attribute values and/or incident edges, \mathbf{b}_u and $\tilde{\mathbf{b}}_u$ capture the attribute and incident edge information for nodes *u* and \tilde{u} respectively, and *L* is the Lipschitz constant.

3. Our Framework NIFTY

Next, we describe our framework NIFTY which aims to generate fair and stable graph embeddings by enforcing fairness and stability in the objective function as well as in the architecture of the underlying GNN.

Problem formulation (Fair and Stable embeddings). Given $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{X})$, NIFTY aims to generate embeddings $\mathbf{z}_u \in \mathbb{R}^d$ that are counterfactually fair (Eq. 1) and stable to attribute and structural perturbations of \mathcal{G} (Eq. 2).

Fairness and Stability in the Objective Function. To infuse fairness and stability in the objective function, we introduce a triplet-based objective that maximizes the agreement between the original graph and its counterfactual and noisy views. To this end, we use Siamese networks to maximize this agreement, i.e., the augmented network neighborhoods and attribute vectors of the same node should result in the same embedding (Chen & He, 2020). Generating augmented views of graph structure and attribute information is key for the Siamese learning. We generate them using node-, sensitive attribute-, and edge-level perturbations. Refer Appendix D for details. To learn embeddings that are invariant to the sensitive attribute and stable against perturbations, we train the GNN encoder ENC using the Siamese framework (Bromley et al., 1994) and generate representations $\tilde{\mathbf{z}}_{u}$ of the augmented graph at every iteration. By generating augmented graphs, NIFTY can induce appropriate bias into the underlying GNN to learn embeddings invariant to the combination of counterfactual and random perturbations. A predictor $t : \mathbb{R}^d \to \mathbb{R}^d$ consisting of a fully-connected layer

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is then used to transform and match the representations
with each other. Inspired by Grill et al. (2020), we define a
triplet-based objective that optimizes the similarity between
the graph and its augmented (*i.e.*, counterfactual and noisy)
representations:

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$$\mathcal{L}_{s} = \mathbb{E}_{u} \Big[\frac{1}{2} \big(D(t(\mathbf{z}_{u}), \operatorname{sg}(\tilde{\mathbf{z}}_{u})) + D(t(\tilde{\mathbf{z}}_{u}), \operatorname{sg}(\mathbf{z}_{u})) \big) \Big],$$
(3)

117 where $t(\mathbf{z}_u)$ and $t(\tilde{\mathbf{z}}_u)$ are the transformed representations 118 of u and \tilde{u} , D is the cosine distance, and stopgrad (sg) pre-119 vents gradients from being backpropagated. The sg signifies 120 that the node representation $\tilde{\mathbf{z}}_u$ is considered as constant 121 when operating on $t(\mathbf{z}_u)$ and vice-versa. The overall objec-122 tive function for NIFTY is: $\min_{\theta_{\text{ENC}}, \theta_t, \theta_f} \mathbb{E}_u \left[(1 - \lambda) \mathcal{L}_c \right] +$ 123 $\lambda \mathcal{L}_s$, where $\{\theta_{\text{ENC}}, \theta_t, \theta_f\}$ denotes trainable parameters of 124 ENC, t, and classifier f, \mathcal{L}_c is the binary cross entropy loss, 125 and the expectation is taken over training nodes in \mathcal{G} . The 126 regularization coefficient λ controls the trade-off between 127 node classification loss \mathcal{L}_c and the tripled-based objective 128 \mathcal{L}_s . Algorithm 1 givens an overview of NIFTY algorithm. 129

130 Fairness and Stability in the GNN Architecture. 131 NIFTY modifies the GNN's routing of neural mes-132 sages. Typically, a GNN layer is given by (Sec. 2): 133 $\mathbf{h}_{u}^{k} = \text{UPD}(\text{AGG}(\text{MSG}(\mathbf{h}_{u}^{k-1}, \mathbf{h}_{v}^{k-1}) | v \in \mathcal{N}_{u}), \mathbf{h}_{u}^{k-1}).$ Con-134 sidering AGG a fully-connected neural layer and UPD a non-135 linear activation function σ , we rewrite the message-passing step as: $\mathbf{h}_{u}^{k} = \sigma (\mathbf{W}_{a}^{k} \mathbf{h}_{u}^{k-1} + \mathbf{W}_{n}^{k} \sum_{v \in \mathcal{N}(u)} \mathbf{h}_{v}^{k-1})$, where 136 137 \mathbf{W}_n^k is the weight matrix associated with the neighbors of 138 node u and \mathbf{W}_{a}^{k} is the self-attention weight matrix at layer 139 k. Definition 2 entails that the Lipschitz constant L provides 140 an upper bound on how much u's node embedding can 141 change. In fact, L represents the smallest value for which 142 Eqn. 2 in Def. 2 holds true. Leveraging this understanding, 143 NIFTY bounds the change in *u*'s embedding by normalizing 144 the encoder's weight matrices. This is because of the slope-145 restricted structure of the nonlinear activation function in the 146 UPD step (proof in Sec. 4). Using our derivations in Sec. 4, 147 we calculate the Lipschitz constant L of term $\mathbf{W}_{a}^{k}\mathbf{h}_{u}^{k-1}$ as 148 the spectral norm σ of the weight matrix at each layer k. We use *L* to normalize \mathbf{W}_{a}^{k} as: $\tilde{\mathbf{W}}_{a}^{k} = \mathbf{W}_{a}^{k}/\sigma(\mathbf{W}_{a}^{k})$. We use this normalized weight matrix $\tilde{\mathbf{W}}_{a}^{k} = \mathbf{W}_{a}^{k}/\sigma(\mathbf{W}_{a}^{k})$. We use this normalized weight matrix $\tilde{\mathbf{W}}_{a}^{k}$ to modify the UPD step: $\mathbf{h}_{u}^{k} = \sigma(\tilde{\mathbf{W}}_{a}^{k} \mathbf{h}_{u}^{k-1} + \mathbf{W}_{n}^{k} \sum_{v \in \mathcal{N}(u)} \mathbf{h}_{v}^{k-1})$. Lipschitz normalization has two advantages: 1) it bounds the 149 150 151 152 153 difference between embeddings of original and perturbed 154 node attributes; 2) it establishes a connection between 155 stability and counterfactual fairness such that similar inputs 156 should yield similar representations. 157

4. Theoretical Analysis of NIFTY

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Next, we 1) prove that NIFTY's embeddings are stable, 2) provide a theoretical upper bound on unfairness of embeddings, and 3) show that downstream classifiers trained on NIFTY's embeddings are counterfactual fair.

Theorem 1 (NIFTY Stability). Given a non-linear activation function σ that is Lipschitz continuous, the representations learned by our framework NIFTY are stable i.e.,

$$||\text{ENC}(\tilde{u}) - \text{ENC}(u)||_{p} \le \prod_{k=1}^{K} ||\mathbf{W}_{a}^{k}||_{p}||(\tilde{\mathbf{b}}_{u} - \mathbf{b}_{u})||_{p},$$
 (4)

where \tilde{u} is a node in the augmented graph generated by perturbing *u*'s attribute values and/or incident edges, \mathbf{b}_u and $\tilde{\mathbf{b}}_u$ capture all attribute and edge information of nodes *u* and \tilde{u} , and \mathbf{W}_a^k is *u*'s self-attention weight at layer *k*.

Proof is provided in the Appendix A.

Theorem 2 (NIFTY Counterfactual Fairness). Given a non-linear activation function σ that is Lipschitz continuous and a binary sensitive attribute s, the (counterfactual) unfairness of NIFTY's representations can be bounded as:

$$||\operatorname{ENC}(\tilde{u}^s) - \operatorname{ENC}(u)||_p \le \prod_{k=1}^K ||\mathbf{W}_a^k||_p \tag{5}$$

where \tilde{u}^s is a node in the augmented graph which is generated by modifying (flipping) the value of the sensitive attribute (s) of node u while keeping everything else constant.

Proof is provided in Appendix B.

Proposition 1 (Counterfactual Fairness of Downstream Classifier). If NIFTY's representations satisfy counterfactual fairness, then a downstream classifier $f : \mathbf{z}_u \rightarrow \hat{y}_u$ using those representations also satisfies counterfactual fairness.

Proof is provided in the Appendix C.

5. Experiments

Next, we present experimental results for NIFTY framework. We first describe datasets designed to study fair and stable network embeddings and then outline experimental setup.

Datasets. We construct three new datasets. 1) The German credit graph has 1,000 nodes representing clients in a German bank connected based on the similarity of their credit accounts. The task is to classify clients into good vs. bad credit risks considering clients' gender as the sensitive attribute (Dua & Graff, 2017). 2) The Recidivism graph has 18,876 nodes representing defendants who got released on bail at the U.S. state courts between 1990-2009 (Jordan & Freiburger, 2015). Defendants are connected based on the similarity of past criminal records and demographics, where the goal is to classify defendants into bail (*i.e.*, unlikely to commit a crime if released) vs. no bail (i.e., likely to commit a crime) considering race information as the protected attribute. 3) The Credit defaulter graph has 30,000 nodes representing individuals connected based on the similarity of their spending and payment patterns (Yeh & Lien, 2009).

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Figure 2. Unfairness and instability error rates for five GNNs and their NIFTY counterparts. NIFTY-enhanced GNNs give fairer and more stable predictions than their unmodified counterparts.

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The task is to predict whether an individual will default on
the payment or not while considering age as the sensitive
attribute. See Appendix E for details.

Metrics. We use AUROC and F1-score to measure GNN's 184 185 predictive performance. To quantify group fairness, we use statistical parity (Δ_{SP}), and equal opportunity (Δ_{EQ}) as 186 defined in Dai & Wang (2021). To measure counterfactual 187 fairness, we define the unfairness score as the percentage of 188 test nodes for which predicted label changes when the node's 189 sensitive attribute is flipped. Finally, the instability score rep-190 resents the percentage of test nodes for which predicted label 191 changes when random noise is added to node attributes.

193 GNN methods. We incorporate NIFTY into five GNN methods: GCN (Kipf & Welling, 2017), GraphSAGE (Hamilton 195 et al., 2017), Jumping Knowledge (JK) (Xu et al., 2018), 196 GIN (Xu et al., 2019), and InfoMax (Veličković et al., 2019). 197 Additionally, we consider two baseline methods: FairGCN 198 (Dai & Wang, 2021) and RobustGCN (Zhu et al., 2019); all 199 hyperparameters are set following the authors' guidelines. 200 We use stop-gradient operation for training the Siamese net-201 works (Chen & He, 2020). We set regularization coefficient 202 to $\lambda = 0.6$ and conduct a sensitivity analysis into the effect of λ on NIFTY's performance. See Appendix F for details. 204

Results: NIFTY improves fairness and stability. Across three datasets and five GNNs, NIFTY-modified GNNs learn 206 fairer and more stable embeddings than unmodified GNNs (Fig. 2). On average, NIFTY improves stability and fairness 208 of GNNs by 60.87% and 92.01%, respectively. Further, 209 NIFTY can promote fairness and stability of GNNs without 210 sacrificing their predictive performance, as evident by AU-211 ROC and F1-scores in Table 2. Finally, NIFTY outperforms 212 baseline FairGCN and RobustGCN methods by 62.07% and 213 214 57.26% on four fairness and stability metrics (Table 1).

Results: NIFTY achieves group fairness. While NIFTY
 aims to capture counterfactual fairness, it remarkably improves group fairness of GNNs as it reduces information on
 protected attributes and makes the multi-objective problem



Figure 3. The effects of regularization on the performance of NIFTY-GIN for the German credit graph (see Fig. 4 for other datasets) where NIFTY achieves a near-perfect stability and fairness on the downstream task over a wide range of regularization.

of satisfying fairness and stability more tractable. Across three datasets, five GNNs, and two group fairness metrics, NIFTY achieves 43.56% lower Δ_{SP} and 34.70% lower Δ_{EO} , suggesting that in NIFTY, a node's chance of being represented as a particular point in the embedding space does not depend on its membership in a protected group.

Results: Fairness vs. stability vs. accuracy. As we increase regularization coefficient λ in NIFTY (Fig. 3), we find that the error rates for counterfactual fairness and stability steadily decrease. Even with a modest amount of regularization (λ =0.1), NIFTY achieves a 94.29% improvement in unfairness. As expected, a strongly regularized NIFTY model takes a hit on its predictive performance.

Results: Ablation study. We conduct ablations on two key NIFTY's components, namely the objective function and the Lipschitz layer-wise normalization. Results show that both components are necessary to generate fair and stable embeddings (Table 3). In particular, we observe a 90.7% improvement in fairness of NIFTY-GCN as compared to vanilla GCN, providing empirical evidence for our theoretical analysis that the Lipschitz normalization can improve both fairness and stability of graph embeddings (Sec. 4).

6. Conclusions & Future Work

We propose NIFTY, a unified framework that exploits a novel connection between counterfactual fairness and stability to learn network representations that are both fair and stable. NIFTY uses a two-level strategy to modify an existing GNN at the architectural and the objective function level. Results on new graph datasets from criminal justice and financial lending domains show that NIFTY improves fairness (counterfactual and group fairness) and stability without sacrificing predictive performance. This work paves way for several future directions, *e.g.*, extending NIFTY to generate fair and stable representations of other graph components (*e.g.*, edges, subgraphs) and to cater to other downstream tasks (*e.g.*, link prediction, graph classification).

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